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THE MEASUREMENT AND ANALYSIS OF FATIGUE CRACK GROWTH IN THICK-WALLED CYLINDERS

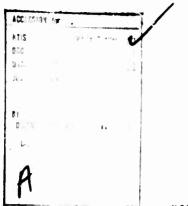
J. J. Miller, et al

Watervliet Arsenal Watervliet, New York

September 1974

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a function of the crack propagation rate. The portion of fatigue life consumed in crack initiation may be considered an additional safety factor. To develop design criteria and to set safe inspection standards, it is necessary to know not only the length, but the depth and shape of fatigue cracks.

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Block No. 20 ABSTRACT(Continued)

A technique has been developed for the accurate measurement of crack depth in thick-walled cylinders using a straight-beam ultrasonic probe. This technique has proved useful not only under laboratory conditions, but also in practical situations such as inspection of cannon tubes after firing. Various aspects of the technique such as accuracy and sensitivity are presented.

In the presence of multiple cracks, the measurement, recording and analysis of all the factors of fatigue crack growth can be quite complicated. To alleviate this problem a system for the rapid collection, reduction, and analysis of this data by computer has been developed. The resulting measuring devices, the data acquisition system and the link to an IBM 360 computer is described.

Various computer programs used to process the data and illustrating the kind of information and graphical output that can be generated are described. This includes a graphic representation of all the cracks present showing their location and depth. The importance of the rapid data analysis feature of the system to apply fracture mechanics theory for the control of the fracture process in thick walled cylinders is demonstrated.

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THE MEASUREMENT AND ANALYSIS OF FATIGUE CRACK
GROWTH IN THICK-WALLED CYLINDERS

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*THE MEASUREMENT AND ANALYSIS OF FATIGUE CRACK GROWTH IN THICK-WALLED CYLINDERS

INTRODUCTION

Cannon tubes are essentially thick-walled pressure vessels. When exposed to repeated cycles of high internal pressure, as they are during firing, they may fail by low-cycle fatigue. Cannon tube failures in the recent past created the need to study this fatigue problem. Because of the high temperatures produced in the cannon during firing, "heat-check" cracks are formed in the bore after firing a very few rounds. The crack initiation period then can be considered over and the fatique life of these tubes becomes a function of the crack propagation rate. Proceeding on this premise, Watervliet Arsenal has conducted a series of laboratory fatigue tests of cylinders cut from cannon tubes (1). The tests consist of firing cannon tubes several hundred times to produce typical heatcheck (incipient crack) patterns. Short cylinders are then cut from the tubes, sealed at each end, and subjected to cycles of internal hydraulic pressure comparable to firing. During the hydraulic pressure cycling the cracks enlarge as they would in firing and are measured at frequent intervals until the cylinder fails.

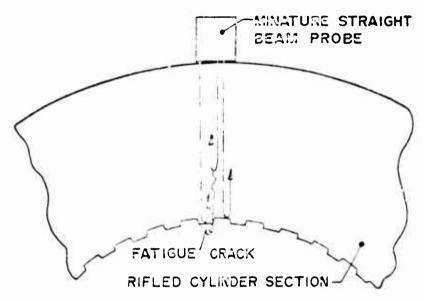
The procedure outlined above is a faster more efficient method than firing alone for establishing fatigue life. However, to make use of the current technology in fracture mechanics, it is necessary to monitor both depth and shape of the fatigue cracks during cycling. In the course of this investigation we have developed an ultrasonic technique to measure crack depth, a locating device and data acquisition system to automatically record findings, and finally, computer programs to analyze and report test results in a useful graphic format. This paper describes and evaluates these three interdependent developments.

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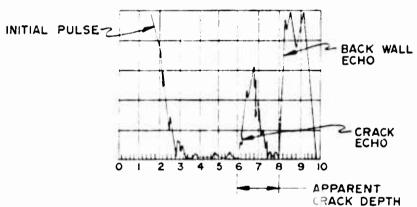
ULTRASONIC CRACK DEPTH MEASUREMENT

A fatigue crack is an excellent reflector of ultrasonic energy when the plane of the crack is nearly normal to the ultrasonic beam. In the case of cannon tubes, cracks grow radially from the bore and can be detected very early with angle probes, but depth measurement with an angle probe on a cylindrical body is very difficult and in the presence of multiple cracks is virtually impossible. A straight beam probe when in contact with the outside surface of the cylinder can also detect a crack even though the crack is oriented "head-on" to the sound beam (2). This arrangement is shown in Fig 1. The narrow target presented by a closed crack would hardly seem sufficient to reflect an ultrasonic signal. On closer inspection, it can be seen that fatigue cracks follow a rather crooked path with many facets sharply inclined to the plane of the crack. In projection the crack is not a narrow line but a band. The crooked path and many facets represent a substantially larger target than expected and make it possible for a crack in this orientation to be detected. Naturally a high gain setting is required to detect cracks with this orientation (approximately 30 dB higher).

This method of inspection, shown in Fig la, has two major advantages. It allows the operator to measure crack depth directly from the ultrasonic scope and also to locate the position of the crack without complex calculations. Because the crack reflects only a fraction of the sound energy, both the crack and the "back wall" echoes appear on the ultrasonic scope at the same time, as shown in Fig 1b. The distance between the crack tip and back wall, in effect the crack depth, appears as a linear distance on the scope. By judicious calibration of the instrument the operator can set the scale to represent decimal fractions of an inch and he can read crack depth directly from the distance indicated on the scope. This measurement is now done electronically as described later. At the same time the operator locates the crack exactly by scanning transverse to its length. When the distance to the crack is minimized the probe is positioned directly over the crack. Thus, with a simple scanning over the outside of the cylinder, the operator is able to locate and measure cracks extending from the inside surface. With repeated transverse scans the operator can locate points on the crack along its length, and measure depths at some chosen interval. By scanning the entire cylinder, a "crack map" is developed. With continued cycling and additional "crack maps" one is able to follow the progress and shape changes that occur in all the cracks in the tube. Without prior knowledge of which crack will cause failure it is necessary to follow all cracks. Later, as a few dominant cracks emerge, data taking can be more selective. When failure finally occurs and one crack penetrates the tube wall, the changes in any particular crack can be traced back through the series of crack maps. From this crack map data one can plot curves of crack depth



a SCHEMATIC OF PROBE-CRACK ORIENTATION



b. SCHEMATIC OF FLAW DETECTOR PRESENTATION

FIGURE I. SCHEMATICS OF THE ULTRASONIC CRACK MEASUREMENT METHOD USED IN THIS REPORT

versus cycles, crack rate versus cycles, crack rate versus relative crack depth, and crack rate versus stress intensity factor for any crack of interest.

ACCURACY OF ULTRASONIC MEASUREMENT

The accuracy of crack depth measurement has been checked by comparing the ultrasonic and visual measurements. When a cylinder is partially fractured in fatigue and then wedged apart, the fracture surface shows a sharp change in fracture appearance. It is then relatively easy to measure the actual depth that the fatigue crack had prior to fracture. By growing fatigue cracks at a relatively high load and then cycling for a long time at a sharply reduced load. which apparently produces no crack growth, one can see on the fractured surface a fine line representing this period of no growth. By producing a series of these no growth lines in a fatigue specimen and measuring crack depth ultrasonically at the same time one is able to make several checks of apparent (ultrasonic) versus actual crack depth in the same cylinder. The results from a series of such cylinders is shown in Fig 2. In this case four cylinders were fatigue tested and the crack depth measured ultrasonically at frequent intervals. It can be seen that the ultrasonic measurement agrees quite well with the visual measurement. The relationship varies somewhat from one type specimen to another but in general the accuracy of this method of ultrasonic crack depth measurement may be expressed as follows:

 $d = (b - 0.03) \pm 0.03$ where

d = crack depth measured by ultrasonics and

b = actual crack depth in inches

The apparent depth (d) is less than the actual depth because the reflecting surfaces are behind the crack tip.

AUTOMATED CRACK POSITION AND DEPTH MEASUREMENT

Until recently, crack maps have been made manually by marking the tube surface with a crayon indicating the depth of the cracks at each point. Then using a protractor and measuring stick the operator has located these cracks according to their angular position θ around the cylinder and longitudinal position Y along its length and manually recorded this information using rectangular coordinates on a graph or "crack map". The coordinates are θ in degrees in the plane normal to the cylinder axis and Y in inches measured from one end of the cylinder.

This method of measurement of crack depth and position is tedious and the accuracy is affected by operator skill, judgment and experience. A semi-automatic measuring and recording system has been designed and built which reduces operator error and

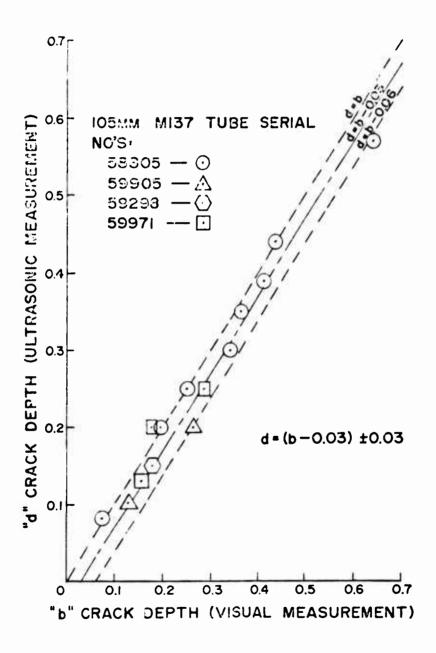


Figure 2. Crack Depth (Ultrasonic) vs. Crack Depth (Visual)

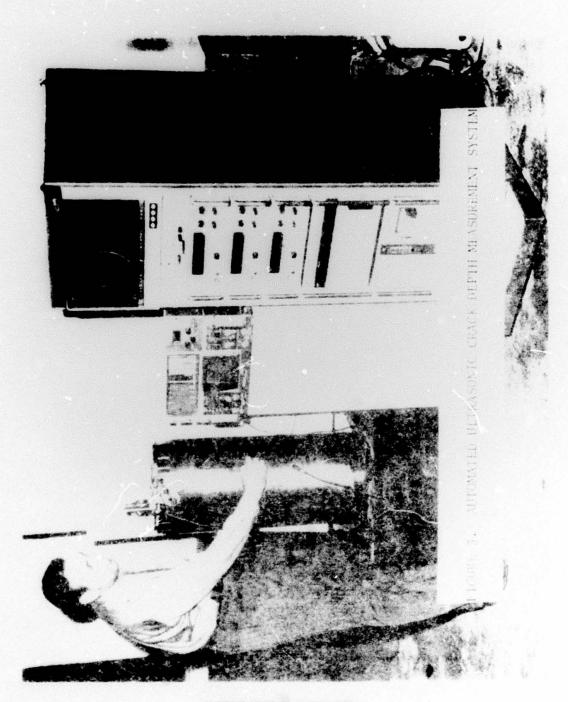
eliminates manual data taking and graph plotting.

The crack position on the thick walled cylinder and its length along the cylinder is described by a two coordinate locator. This device is fixed to the top of the cylinder at a known reference point by two switchable magnets and is attached to the ultrasonic probe by a flexible steel cable as shown in Fig 3. When the probe is moved over the surface of the cylinder in the process of looking for and measuring cracks the locator ragisters the position of the probe. This position is described by length ℓ of the flexible cable from the reference point to the probe, and by the angle ℓ between the cable and a reference line in the plane tangent to the cylinder and containing the reference point. We usually mark a second reference point at the bottom of the cylinder and directly under the first so that we can calibrate to the known length to this point and known angle (ℓ = 90°).

The two encoders used to register ℓ and ℓ in the locator are Gurley Model 8610 incremental rotary encoders. These were chosen because of their small size and low static friction. Rotation of the disc in these encoders produces quadrature pulses from two phototransistor circuits. When connected to a reversible counter, the count is positive for one direction of rotation and negative for the other. The counters can be reset to zero at any shaft position of the encoders and rotation will be measured in either direction from this position. The ℓ encoder has a standard disc which provides 250 pulses per revolution. The pulley configuration for the steel cable permits four turns in 10 in. or one pulse per .01 in. of cable travel. The ℓ encoder has a standard disc with 360 pulses per revolution. Since the plotter is usually located at the top of the cylinder, we only use this encoder over a half revolution (180°) of the shaft, or one pulse per degree of rotation.

The errors in these measurements are 0.15 in. maximum, in length, ℓ , and ℓ 1° in angle, ℓ . This is 0.5% for a 30 in. cylinder in ℓ and 0.5% for ℓ (1/180). The length error is attributed to an hysteresis effect in the locator. The flexible cable is reeled on a drum actuated by a constant torque spring. Due to friction losses the torque is greater when pulling the cable out than when reeling it in, causing more cable stretch going out than in. As a result, after mapping a half cylinder the last reading may have an accumulated error as much as 0.15 in. The error in ℓ is due primarily to the quantization in the counters (ℓ one count).

For a 1° error in \mathcal{D} , the error in specifying a crack position on the crack map becomes a function of the distance from the locator reference point. For this reason, and because of a slight droop in the cable, the locator is used to map half the cylinder from the θ = 0 reference point and is then moved to θ =180° to map the other half.



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The electronic system developed for automated depth measurement, is shown in Fig 3 (top center). It measures the time interval from the time the sound pulse enters into the cylinder until it reflects from the nearest crack tip or the bore surface and returns to the probe. Sound travels .02 in. in 86.6 nanoseconds in steel. This is .01 in. towards the reflector and .01 in. back for each 86.6 nanosecond interval. A three decade counter counts the number of oscillations from an oscillator (clock) whose period is 85.6 nanoseconds. This depth reader is synchronized to the ultrasonic flaw detector so that its clock and counters are reset to zero and gated on at the time that the transmitted pulse from the flaw detector enters the cylinder. The counters count from zero to 999 for each transmitted pulse. The signal echoes from crack tips are connected to a variable threshold circuit in the depth reader. All signals above the threshold level provide an output to a coincidence gate. Noise signals below the threshold cause no output. The first gate output following the transmitted pulse causes the number in the decade counter at that instant to be transferred to an output register. This number is the depth in hundredths of an inch from the surface of the cylinder to the reflector at that point. The maximum wall thickness to be measured is less than 5 inches, therefore a three decade counter and output register which permit readings up to 9.99 inches is adequate. The accuracy of the decade counter circuits is \pm one count or \pm .01 in. The overall accuracy of the depth reader is ± .03 in.

The gated synchronized oscillator frequency can be varied by means of a coarse and fine frequency control to calibrate for the velocity of sound in different materials. This oscillator is called synchronous because its first pulse and all succeeding pulses bear a fixed time relationship to the leading edge of the square wave gate signal that turns it on and off. The time when the counter starts can be varied relative to the transmitted pulse, thus providing a correction for one point on a linear calibration plot of true depth vs apparent reading depth, while the frequency control varies the slope of such a line.

The circuitry used in the crack depth reader is made from commercially available integrated circuits. The three decade character (digit) display uses compatible Light Emitting Diodes. The digital time interval measurement technique for obtaining crack depth was chosen in preference to analog techniques because it could be implemented using integrated circuits and because the following logic operations could be performed. Data is transferred only if: (1) the operator's hand or foot operated switch is in closed (read) position; (2) the reading is not on any part of the transmitted pulse; (3) the reading is not on the back wall or bore surface (optional); (4) previous data is not being transferred to the tape recorder. Furthermore, the transfer pulse will be inhibited until after the next initial pulse if the operator read command occurs

between the initial pulse and the back wall signal. This avoids an erroneous reading whose chance of occurring is 1 in 6 at the maximum ultrasonic flaw detector repetition rate of 2000 Hz.

The crack depth number in the output register of the crack depth reader is in binary coded decimal form. It provides depth data to the Data Acquisition System (DAS) whenever the operator pushes the "read" switch. The outputs of the \$\mathcal{L}\$ and \$\eta\$ encoders are a number of pulses proportional to the angles through which the encoder shafts have turned. Hewlett Packard 5280A Reversible Counters operate directly from these encoders without requiring preamplification of the encoder output pulses. The counters count these pulses and continuously present the total count in binary coded decimal form to the DAS. The counters have an output buffer register which permits a read-out without interrupting the count.

DATA ACQUISITION SYSTEM

A Hewlett-Packard Coupler Model 2547A is used to process the parallel binary coded decimal data from four sources. The L, B and depth sources have been described above. The fourth source is the bank of 12 manual data thumb wheel switches on the front panel of the coupler. The bainary coded decimal outputs of these tenposition switches are used for identifying the data being recorded on magnetic tape.

These four sources provide all data simultaneously to the Coupler. The Coupler translates this data to serial form, formats it, and feeds it to an incremental magnetic tape recorder. The Coupler is, in effect, a very small special purpose data processer. It has input buffer registers which store the incoming data on command from a manual foot switch; a control circuit that provides a hold off signal to the data sources; a serializer that steps thru the required number of steps for the number of digits in use and provides an inter-record gap (in our case after every word); a word format patchboard by which we can select both the order in which characters are recorded from a source and the length of the recorded word; and an output buffer which connects to the magnetic tape unit input.

In addition to the output buffer, the Coupler has a secondary buffer which retransmits the input data, in its original form, to a Hewlett Packard 5050B digital recorder which furnishes a paper strip printout. This provides a visual check on the data being recorded on magnetic tape. This recorder can print 360 characters per second, so it does not greatly restrict the tape recording speed which is a maximum of 500 characters per second.

The magnetic tape unit is a 7 track IBM compatible Kennedy Model 1406. It holds a 1200 foot reel of 1/2 inch magnetic tape.

The tape unit has its own circuits for inserting standardized gaps into the data from the Coupler which can be read by the IBM 360/44 computer.

DATA ANALYSIS

When the crack data recording has been completed the seven track tape is transferred to the computer. The computer program "ULTRARAX" checks the accuracy of the measurement system in order to insure that no malfunction of the locator has occurred. If there is a malfunction causing inaccurate measurement of the reference points on the cylinder, then the program aborts and indicates where the inaccurate measurement(s) took place for troubleshooting purposes. If the measurements are all acceptable the program accepts the data, translates from a cylindrical to a rectangular coordinate system and constructs the crack maps on the high-speed printer as illustrated in Fig 4. The original measured values are also tabulated by the high speed printer.

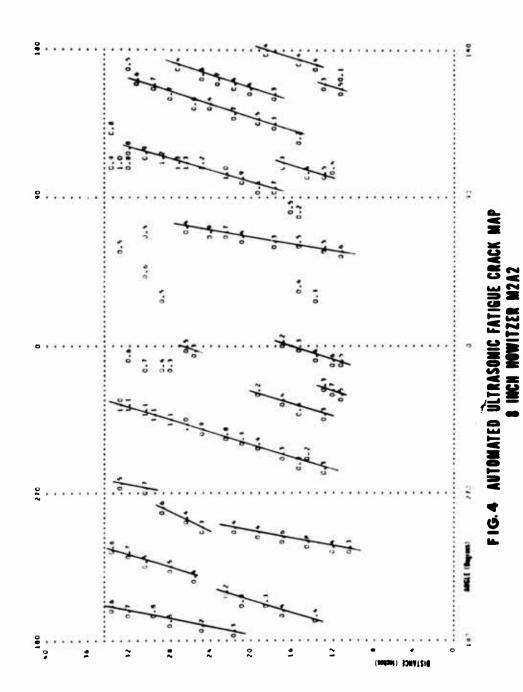
The crack map shown in Fig 4 displays the location and depth of several cracks in a representative cylinder. The abscissa represents the coordinate, θ , in degrees around the cylinder. The ordinate represents distance y from the bottom of the cylinder in inches. The measurements registered in the crack locator in ℓ and β coordinates are transformed to the θ , y coordinates by the following equations:

$$x = l \cos \theta$$
; $y = L - l \sin \theta$

$$\theta = \frac{x}{R} = \frac{\ell \cos \theta}{R}$$

where R is the outside radius and L is the overall length of the cylinder. The numbers on the plot indicate the crack depths in nearest tenths of an inch at their specified locations. The decimal point is plotted at the location of the measurement. Since the high-speed printer is limited by a discrete number of characters on a line (10 per inch) and lines in a column (6 per inch), all crack measurement locations must be rounded off within these limitations for plotting purposes. After each testing interval ultrasonic measurements are made and recorded and the crack mapping is repeated. Each crack map along with its data is recorded, processed, stored and indexed on a 9-track magnetic tape devoted to this data analysis.

Once the cylinder has failed and all crack maps have been recorded the computer program "ULTOCRAK" scans the crack maps stored on the 9-track tape at each measurement interval and chooses the maximum depth of the dominant crack, or the crack of interest, for each successive crack map. A window technique, limiting the scan to a range of 0, is used to screen all other unwanted data. The data thus selected becomes fatigue crack growth data for the computer



program "CRACKERS".

The "CRACKERS" program utilizes a least-squares point-to-point spline-fitting technique (3)(4) to express equations which describe the fatigue crack growth and enable evaluation of the crack propagation rate throughout the cyclic life of the cylinder. The name "spline function" is derived from the fact that a third-degree spline function approximates the behavior of a draftsman's mechanical spline. In the "CRACKERS" program each data point contributes equal weight, and the spline segments involved are from point-to-point. The equation for each of these segments permits mathematical differentiation to obtain the crack propagation rate, which is the first derivative of crack depth with respect to number of fatigue cycles.

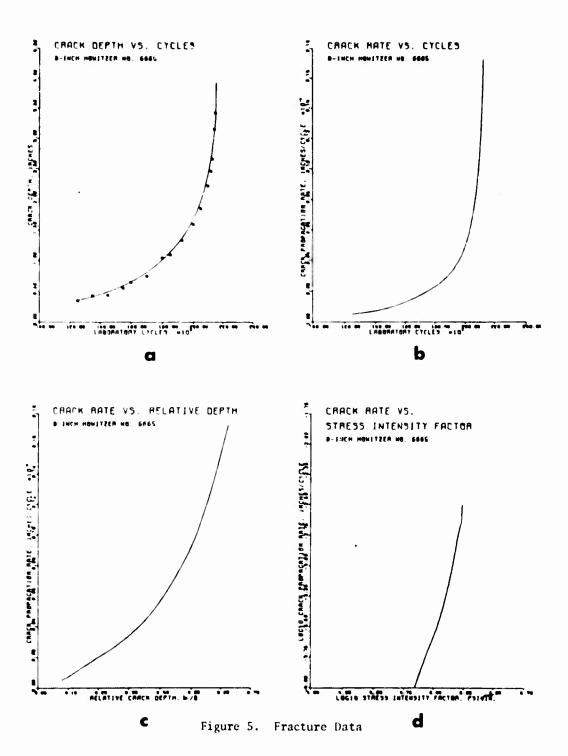
From the fatigue data on a particular crack, obtained by scanning the crack maps—and from the spline-generated results, the computer program "CRACKERS" also plots several other graphs on the high-speed printer. Because of the discrete plotting-space printer, the printer plotting is used only as an intermediate step to indicate the trend of the plotted data. Since the spline-generated results allow for a continuous curve to be plotted through the crack growth data these results are transferred to a magnetic tape as input data to a Cal-Comp x-y plotter. This process has the advantage that the graphs from the plotter are in finished form.

Fig 5 illustrates typical plots from the "CRACKERS" program. Curve 5a is a graph of crack depth, b, versus number of cycles, N. The plotted points represent measured crack growth data obtained from the crack maps and the line curve represents the results generated from the spline function. Figs 5b, 5c and 5d are based on results generated from the spline function equations, so they are represented only as line curves. In Fig 5b the curve represents crack propagation rate, db/dN, versus number of cycles, N, obtained by the differentiation of curve 5a. In Fig 5c the curve is a plot of crack propagation rate, db/dN, versus relative crack depth, b/B, while the curve in Fig 5d shows a log-log plot of crack propagation rate, db/dN, versus stress intensity factor, K, where the value of K has been calculated from an approximate formula for a longitudinal wall crack in a cylinder.

DISCUSSION

The technique and system described earlier, leading to the above typical computer analysis, supplies that information necessary for the application of fracture mechanics to the problem of fatigue and fracture of thick walled cylinders.

The value of material fracture toughness K_{IC} and the characteristic relation between crack rate and stress intensity factor may be evaluated with material test specimens. The crack



growth in a given cylinder may then be predicted by calculation of the critical crack depth and the number of cycles for propagation from an initial flaw size to failure at the critical crack size. Thus the computer aided analysis of crack growth data enables the rapid interpretation of the fatigue behavior and estimation of the performance to be expected from a given cylinder or component. By the addition of a decision program it should be possible for a computer to estimate remaining fatigue life. These programs, together with an automatic scanner for collecting the ultrasonic data, could form the basis for a feasibility model of an automatic field cannon tube evaluator.

CONCLUSIONS

- 1. The straight-beam probe technique reliably measures the depths of radial fatique cracks in thick walled cylinders and eliminates the limitations of the angle-beam approach.
- 2. Operator time and interpretation skill requirements in ultrasonic measurements have been reduced by an automatic spatial location and depth reading system.
- 3. Output of the automated system permits direct computer reduction and analysis of the crack data.

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